ANAXAGORAS AND ARISTOTLE ON THE SUN AT THE HORIZON

And at the rim of the horizon a fine, miraculous line separated air from water. A miracle it seemed: straight and yet curved, sharp and yet indeterminable, visible and yet untraceable.[1]

Introduction: the quarrel about the shape of the earth

Most Presocratics, among them Anaximander, Anaximenes, Anaxagoras, and Democritus, believed that the earth is flat and shaped like a drum. In *De Caelo* 293 b 34 ff. Aristotle argues with those who maintain that the earth is flat, until he finally, in 298 b 20, considers the case for a spherical earth as settled. Reading these pages one can still feel how vehement and how complicated the discussion must have been. As nobody was able to go up into outer space and see what the earth really looks like, one had to take refuge to arguments, varying from common sense to the evidence of the senses, alleged physical laws, and even metaphysics. As a matter of fact, the defenders of a flat earth seemed to have common sense on their side: what would prevent us, and especially our antipodes, from falling off a spherical earth? A particular difficulty was that sometimes both parties shared their presuppositions. One party argued, for instance, that the earth must be flat because it is at rest, motionless at the center of the cosmos, all the other celestial bodies circling around it (294 a 10 and 294 b 13 ff.). The idea was, obviously, that a spherical earth could easily roll away, as Simplicius remarks.[2] This argument was a tough one, as the defenders of a spherical earth generally shared the supposition that the earth is immovable at the center of the cosmos.

In this article I will discuss more thoroughly another argument, in which the defenders of the flatness of the earth appeal to a fact from sense experience. I intend to show that the argument is more

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[1] Frederik van Eeden, *De kleine Johannes* ["*Little John*"] ('s-Gravenhage 1887), end of ch. IV (translation Liesbeth van der Sluijs).

[2] *Simplicii in Aristotelis* De Caelo *Commentaria*, ed. I. L. Heiberg (Berolini 1894) 520, 13–14.

sophisticated than it seems to be at first sight. In trying to understand what is meant I will refer to Simplicius' sometimes, although unfortunately not always, elucidating commentary. Thereupon I will discuss Aristotle's counter-arguments, one of which will prove to be unsatisfactory, and the other correct.

Anaxagoras' argument for a flat earth

In Aristotle's rendition the argument that, as Dmitri Panchenko has shown, must have been used by Anaxagoras,[3] sounds like this: "Some think it (sc. the earth) spherical, others flat and shaped like a drum. These latter adduce as evidence the fact that the sun at its setting and rising shows a straight instead of a curved line where it is cut off from view by the horizon, whereas were the earth spherical, the line of section would necessarily be curved".[4] Aristotle sets himself a rather modest task, which is to show that "this phenomenon (...) gives (...) no cogent

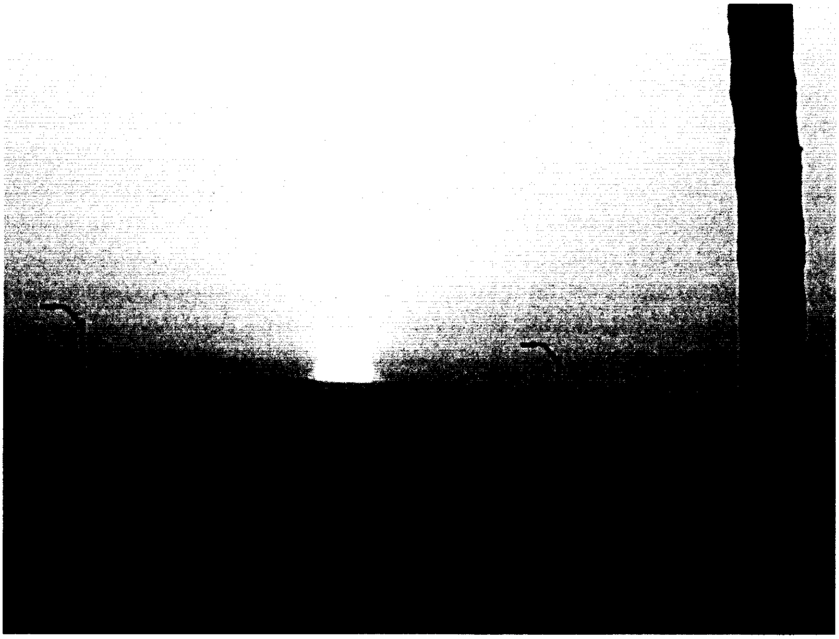


Fig. 1. Sunset at San Diego (author's photograph)

[3] D. Panchenko, "Anaxagoras' Argument Against the Sphericity of the Earth", *Hyperboreus* 3 (1997): 1, 175–178.

[4] The translations of Aristotle's texts are from W. K. C. Guthrie (ed. and transl.), *Aristotle. On the Heavens* (London 1986), unless otherwise indicated.

ground for disbelieving in the spherical shape of the earth's mass" (294 a 8). Nevertheless, as we will see, he has some difficulties in refuting the argument. At first sight the argument looks rather peculiar, and it is even not quite clear what could be meant. In an attempt to understand it, I will start to explore the notion of "horizon".

The meaning of "horizon"

The word "horizon" may mean different things. The *true horizon* is the apparent line that separates earth from sky. Usually, however, we do not see the true horizon, but the *visible horizon* (or the *skyline*), which is the intersection of earth and sky as it is obstructed from free view by trees, buildings, mountains, and so forth. The word "horizon" may also mean: the *astronomical horizon*, which is the horizontal plane through the eyes of the observer. And finally, in perspective drawing the horizon is the straight line towards which parallel lines converge. Although all these meanings of "horizon" will be used, when in the next pages I will speak of the horizon without further indication, the *true horizon* will be meant.

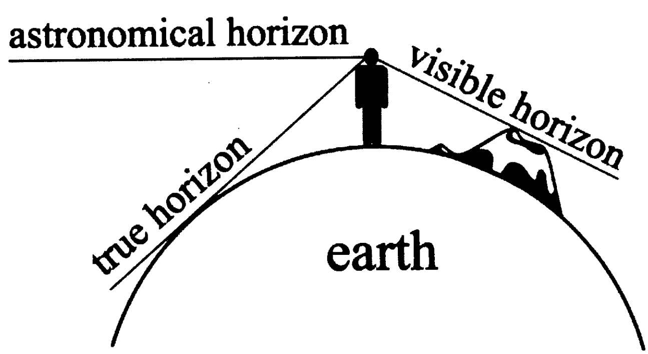


Fig. 2. The true horizon, the visible horizon, and the astronomical horizon (not to scale)

The horizon on a spherical earth

The best way to think about the horizon is to imagine being at full sea, when the weather is bright and I have an unhindered view all around, so that I can see the true horizon. When I turn around my axis for the full 360^0, I will recognize that the horizon makes a full circle around me. The horizon is a visual phenomenon resulting from my eye being above the earth. The horizon determines the relatively small part of the world that is visible to me. When my eyes are at 1.7 meters above the surface, the distance to the horizon is about 4.66 kilometers, according to the formula: $d = \sqrt{h(2r+h)}$, where d is the distance to the horizon, h the height above sea level, and r the earth's radius.[5] If my eye were on the ground, there would be no horizon, or in other words, in the case of zero height the distance to the horizon would be zero as well. Standing at sea level, I am the top of a cone, the length of its side being 4.66 km, and its base being the circular plane of the horizon. As the surface of the earth is curved, the plane of the horizon cuts the earth. At my height of 1.7 m this is also 1.7 m under my feet.[6] This means that the height of the cone with my eye at top amounts to 3.4 m. The circle of the horizon is shown in Fig. 3 (which is also not to scale).

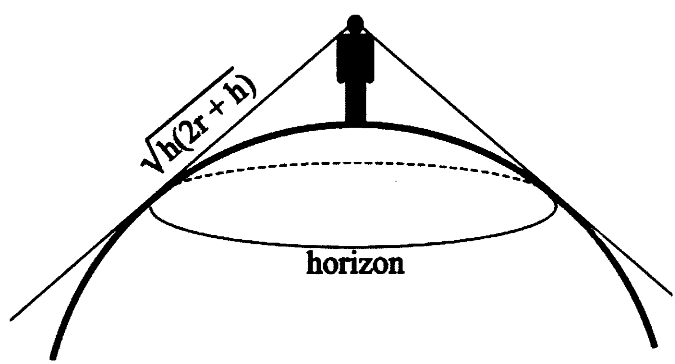


Fig. 3. The circle of the horizon

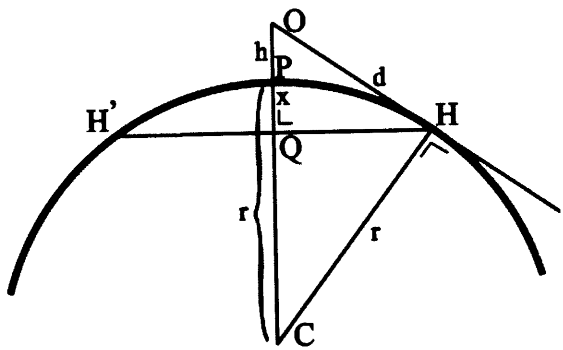
[5] See http://newton.ex.ac.uk/research/qsystems/people/sque/physics/horizon/, where also a calculator for the distance to the horizon at different heights can be found. For the derivation of the formula see note 6.

[6] The derivation of the formula for the distance d to the horizon is found by Pythagoras' theorem. In $\Delta\ OHC$, where HH' is the plane of the horizon, C the center of the earth, O the observer's eye, P the point where the observer's feet stand on the ground, h the height of the observer's eye above the ground, r the earth's radius, x the distance from the observer's feet to the plane of the horizon, d the distance from the observer's eye to the horizon, and $x + QC = r$. Then we get

$$d = \sqrt{(r+h)^2 - r^2} \Rightarrow d = \sqrt{r^2 + 2rh + h^2 - r^2} \Rightarrow d = \sqrt{2hr + h^2} \Rightarrow d = \sqrt{h(2r+h)}$$

When we insert the radius of the earth (6378 km) for r, and the height of the observer (0.0017 km) for h, then $d \approx 4.66$ km.

When I stand still, I see only part of the horizon, which I perceive as a straight line separating earth and sky. Now let us make a little thought experiment. Suppose that I am 100 meters tall (or at a height of 100 m above sea level). Then the distance from my eye to the horizon amounts to 36 km. And when I imagine being 10 km tall (or being in an airplane at that height), the distance to the horizon is 357 km. Growing taller and taller, at a certain moment I will see the horizon clearly as curved, and finally I will be able to see the whole circle of the horizon in one view. Being tall enough, or far enough away, the horizon will practically coincide with the circumference of the earth. When I, at a certain height, see the horizon as curved, I will also observe that the rising sun shows a curved line where it is cut off from my view by the horizon. Now I may conclude that this line always has been curved, although it was not discernable for me as I was too small (or too near the earth's surface). This is actually the same thought experiment as that which Simplicius has in mind, reaching the same result: "Perhaps one should say that if we were outside the earth and saw the sun partially obstructed by the earth, the sections would always appear to us to be curved".[7]



We may find x (the distance from the observer's feet to the plane of the horizon) by comparing the two similar right-angled $\Delta\Delta$ *CHO* and *OQH*.

Then we get

$$(r+h):d = d:(h+x) \Rightarrow d^2 = (r+h)\times(r+h).$$

When we now insert the values 6378 km for r, 0.0017 km for h, and 4.66 km for d, then we get $x = 0.0017$ km.

I wish to thank Ruud Harmsen and my brothers Jan and Leendert Couprie for their kind help with the calculations.

[7] Simplicius (n. 2) 519, 32 – 520, 2, translation by I. Mueller, *Simplicius on Aristotle's "On the Heavens" 2. 10–14* (Ithaca, New York 2005) 59.

The horizon on a flat earth

All this holds for the horizon on a spherical earth. However, Anaxagoras' argument starts from a horizon on a flat earth. On a flat, drum-shaped earth we can do the same thought experiment as we made above for a spherical earth. When Anaxagoras would imagine himself growing to 100 meters and finally to many kilometers above his flat earth, at a certain moment he would observe the curvature of his horizon, and finally he would see the full circular surface of his drum-shaped earth, the horizon coinciding with the full circle of the earth's rim. And he would also see this surface cut the rising or setting sun with a curved line. In other words, what he would see would not differ much from what I saw growing on a spherical earth. He would have been forced to conclude, then, that on a flat earth the sun is cut off at the horizon by a curved line as well, although we see it as straight. In other words, it seems that Anaxagoras' presupposition that the rising or setting sun on a flat earth is cut off by a straight line is easily shown to be false, being an optical illusion, just like in the case of a spherical earth.

We may wonder, however, why Anaxagoras took refuge to a visual phenomenon of which he knew that it was an optical illusion (the Greek text has φαίνεται) that turns out to be the same whether the earth is flat or spherical. This would have been a poor argument for someone who was called "Brains" by his contemporaries (59 A 15 DK). Moreover, it would hardly be understandable why Aristotle paid so much attention to so childish an argument. And finally, Vitruvius (*De arch.* 7, prol. 11) explicitly mentions that Anaxagoras wrote about perspective drawing, where he had to do with another concept of the horizon, as mentioned above. All this is to say that Anaxagoras cannot have meant something trivial with his argument that the sun is cut of by the horizon with a straight line.

The meaning of Anaxagoras' argument

It is again Simplicius, who explains what Anaxagoras probably meant: "*if the arc of a circle is placed in the same plane as the eye*, it will appear to be a straight line, as has been proved in the *Optics*".[8] Mueller notes that he is hinting at Euclid's *Optics*. There we read in theorem κβ' (22): "If the periphery of a circle is placed in the same plane as that of the eye, then the line appears to be straight".[9] Sim-

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[8] Simplicius (n. 2) 519, 22–23. Translation Mueller (n. 7) 60 (my italics).

[9] Mueller (n. 7) 115 n. 283. See I. L. Heiberg (ed.), *Euclidis Optica, Opticorum Recensio Theonis, Catoptrica, cum Scholiis Antiquis* (Lipsiae 1895) 32 (my translation).

plicius identifies the "plane of the circle" with the plane of the horizon: "the horizon is the plane extended through the surface of the earth and our eye (...) the circle which is in the same plane as our eye is seen as a straight line".[10] Strictly speaking, what Simplicius here refers to is the *astronomical* horizon (as explained above), and not the true horizon, which is the result of my being somewhat above the plane of the horizon. The obvious idea is that *practically speaking* I am in the plane of the horizon. Euclid lived about a one and a half century after Anaxagoras. However, Anaxagoras, as said, wrote on perspective drawing, where all parallel lines converge in one point on the straight line of the horizon. This presupposes the idea of an infinite horizon, as parallel lines cut each other in the infinite. So he could very well have pondered about the circumference of an infinite circle (the astronomical horizon) being a straight line as well. If Simplicius is right, Anaxagoras' argument was more subtle than we supposed it to be. In that case he would readily agree what we would see from far above the earth, but he would stress that the argument is not about what we see when we are far above the earth, but about what we see while standing on the earth.

We may use the same thought experiment in the opposite direction to show what is meant. When we are far above Anaxagoras' drum-shaped earth, we see the small circle of its circumference. When we come nearer, this circle grows bigger and bigger, until we cannot see the complete horizon in one view. Finally, when we are in the plane of the horizon, which is the surface of the flat earth, the circle of the horizon is infinite. And an infinite circle equals a straight line. Mark that, whereas my eye on the ground of a spherical earth would result in a zero horizon, as we saw, on a flat earth it would result in a straight line. In other words, Anaxagoras was applying a sophisticated mathematical principle to the phenomenon of the horizon. The gist of his argument can be formulated as follows: "Although strictly speaking we are not in the same plane as the earth's flat surface, but some 1.7 m above it, practically speaking we are in that plane, as our height is negligible in regard to the size of the earth's surface. Consequently, for the horizon on a flat earth holds the same as for the circumference of a circle that is placed in the same plane as the eye. Thus on a flat earth the horizon is seen as a straight line. On a spherical earth, on the contrary, the eye is not in the same plane as the horizon, which is proven by the fact that if, on

[10] Simplicius (n. 2) 520, 6–7, Mueller (n. 7) 60.

a spherical earth, my eye were on the ground, there would be no horizon at all".

Aristotle's refutation of Anaxagoras' argument

Let us now return to Aristotle's text. Aristotle has some pains in refuting the argument that the horizon cuts the rising or setting sun in a straight line, which according to Anaxagoras proves that the earth is flat. Having analyzed what this argument probably really was about, we may understand better what his problems were. What Aristotle tried to do is to show that on a spherical earth the setting sun must be perceived as cut off with a straight line by the horizon as well. In other words, he must make clear that, after all, an optical illusion, and not the mathematics of an infinite circle, is at stake. If he were successful he would be able to conclude that "this phenomenon (...) gives (...) no cogent ground for disbelieving in the spherical shape of the earth's mass" (294 a 8).

Aristotle has two counter-arguments, which are somewhat mixed up in the text. The first is that those who say that the horizon cuts the rising sun with a straight line "fail to take into consideration (...) the distance of the sun from the earth" (294 a 5). In 294 a 7 Aristotle adds some words, which Guthrie in a rather puzzling translation renders as: "(they fail to take into consideration) the appearance of straightness which it naturally presents when seen on the surface of an apparently small circle a great distance away". The Greek text has: ὡς ἐν τοῖς φαινομένοις μικροῖς κύκλοις εὐθεῖα φαίνεται πόρρωθην. Simplicius is not helpful here, for the way he renders these words distorts their meaning: "or that the circles in apparently small bodies appear to be straight lines from a great distance" (καὶ ὅτι οἱ ἐν τοῖς μικροῖς φαινομένοις σώμασι κύκλοι ἀπὸ πλείονος διαστήματος εὐθεῖαι φαίνεται). And the way he explains them is beside the point: "For spherical surfaces are judged to be plane from far away, as in the case of the sun and the moon".[11] A more literal translation of Aristotle's text would sound: "such as in the case of circles that appear to be small: a straight line appears when they are seen from far". I think this translation provides a better understanding of what Aristotle meant: the "little circles" that are "seen from far" are those of the rising and setting sun, and the "straight line" is that part of the horizon that cuts these little circles. Thus translated these added words relate back to Aristotle's first counter-argument,

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[11] Simplicius (n. 2) 519, 26–29, Mueller (n. 7) 60.

which is about the sun being far away. Obviously, he means that the apparent diameter of the sun at the horizon is too small to see the cutting line as curved. For a very small part of a curved line will be seen as straight. In other words, the optical illusion of a straight line cutting the setting sun is due to the relative smallness of the sun disk in regard to the horizon. If the sun were closer to the earth, while keeping its absolute size, it would look bigger. We can imagine the apparent diameter of the sun being so big as to cover, for instance, one third of the visible horizon. Then we would see, Aristotle would argue, that the line that cuts the rising or setting sun is curved. However, this argument is not convincing, as this line would still be seen as straight, even if it would cut a huge rising or setting sun. For, as we saw in the thought experiment, we do not perceive the curvature of the horizon, unless we are far above the surface of the earth. On our height the horizon looks like a straight line, and so it does where it cuts the sun, however big the sun might be.

Aristotle's second argument, separated from the first by the word καί, is that those who say that the horizon cuts the rising sun with a straight line "fail to take into consideration (...) the size of the earth's circumference" (294 a 6). This argument is basically sound. Instead of imagining myself to grow, as I did above, I may also imagine the earth on which I stand to shrink. Then, too, at a certain moment I will see the horizon as curved, or even as a full circle. This is, apparently, what Aristotle has in mind: if the earth were small enough in relation to my length, I would be able to see the horizon as curved. In other words, the optical illusion of the horizon cutting the sun with a straight line is due to the fact that I am so small that I am almost in the plane of the horizon of a big spherical earth. We might reformulate Aristotle's argument as follows, also elaborating it somewhat: "In order to understand the phenomenon of a horizon, the line that separates earth and sky, it is essential to recognize that this phenomenon is due to the fact that our eye is at some distance above the earth's surface. Therefore the issue of the horizon is not the mathematics of an infinite circle with our eye in the plane of that circle. Our eye has to be always at some height above the ground for there to be a horizon at all. This is the same for a flat and for a spherical earth. If our eye were on the ground there would be no horizon. On a spherical earth this can be expressed by saying that the horizon would be zero, and on a flat earth by saying that the horizon would be an infinite straight line. Practically speaking, the height of our eye above the plane of the horizon is almost the same on a flat and on a spherical earth. On a flat earth, where the plane of the horizon coincides with the surface of the earth, the height is 1.7 m at sea level, whereas on a spherical earth this is 3.4 m.[12] These distances are so tiny in regard to the circle of the horizon, that the visual result in both cases is the same: the horizon is perceived as a straight line. The result is that *the issue of the shape of the earth cannot be settled by the argument of the sun at the horizon.* Or in other words: the straightness of the horizontal line cutting the sun 'gives them no cogent ground for disbelieving in the spherical shape of the earth's mass' (294 a 8)".

Appendix: would there be a horizon at all on a flat earth?

Finally, we can speculate on what the phenomenon of the horizon would be on a flat earth like that of Anaxagoras. We are acquainted with the horizon as the limit of our view of the earth, due to the curvature of the earth's surface. Standing on a flat earth, however, we would be able to look much further than the lousy 4.66 km that limits our visual field on a spherical earth. If Anaxagoras' earth were completely flat, without mountains, trees, houses, and so forth, and without any atmosphere that could reduce the range of our view, the horizon would coincide with the rim of the earth. Anaxagoras' earth, however, is not completely flat, but has mountains, etc. Moreover, atmospheric and weather conditions would prevent us from seeing until the end of the flat earth. In other words, on a flat earth only the visible horizon could be perceived, at different distances, depending on seeing conditions. The true horizon of a flat earth would always remain imperceptible. At full sea on a flat earth, and under favorable weather conditions, we must be able to see as far as 50 km or even more. However, because of the atmospheric conditions, on a flat earth the horizon would always appear to be vague, and never so sharply cut as it sometimes is under favorable conditions at 4.66 km distance on our spherical earth. For the same reason, on a flat earth the rising or setting sun would never be perceived as cut at the horizon by a straight line, but always by a blurred and vague line. Perhaps if Anaxagoras had realized how small the distance to the horizon really is, he would have recognized that the simple fact of the existence of the horizon, and especially the sometimes sharp ho-

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[12] As is well known, Aristotle in *de Caelo* 298 b 16 estimated the circumference of the earth too big (400 000 stadia, which equals 63 000 km). Using the calculations of note 6 above, the horizon on an earth of this size would be at about 5.8 km distance, and the plane of the horizon would cut the earth 1.7 m under my feet as well.

rizon, proves that the earth is not flat, but curved, convex, or even spherical.[13]

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Довод Анаксагора в защиту плоской земли – при восходе и заходе Солнца скрытая его часть отсекается прямой, а не дугообразной линией – является более изощренным, чем может показаться на первый взгляд. Как следует из Симпликия, речь шла о математическом подходе к окружности бесконечного круга. Аристотель же сводит рассуждение к оптической иллюзии. При этом можно показать, что первое возражение Аристотеля, указывающее на удаленность солнца, не убедительно, тогда как второе, подчеркивающее величину Земли, правильно.

[13] I wish to thank the members of the Utrecht Study group for the History of Astronomy (USHA), with whom I had the opportunity to discuss some ideas of this article.