Hyperboreus 4, fasc. 2 (1998) 121–149.

Some Notes on Philolaus and the Pythagoreans*

1. Philolaus and the Pythagorean Tradition

In the history of Presocratic philosophy Philolaus was for a long time almost as controversial a figure as the founder of the Pythagorean school himself. But in contrast to Pythagoras, who, as we know, wrote nothing[1], the debate on Philolaus concentrated on the question of the authenticity of his fragments. During the 19th and the first half of the 20th century the majority of scholars took the fragments attributed to him as inauthentic; attempts to prove the contrary, which were not infrequent at this time, did not have much success. As W. Burkert subsequently showed[2], the problem was that among the fragments attributed to Philolaus only a small part could be acknowledged as genuine while the rest clearly bear traces of the later (e.g. Platonic and even Neoplatonic) ideas and terminology foreign to the Presocratics. Thus, it was possible to "save" Philolaus' fragments only by sacrificing a part of them.

The principal conclusions of the famous book by Burkert, who was the first to divide the fragments into two unequal parts and to prove convincingly the authenticity of one part, were surprisingly quickly accepted by the overwhelming majority of the students of Presocratic philosophy. After Burkert, there were no serious attempts to dispute the authenticity of those fragments that he found genuine (B 1–7, 13, 17 DK), and now there is an *opinio communis* in this field, something that is quite rare in the history of Pythagorean studies. It should nevertheless be said that Burkert "saved" some of Philolaus' fragments but not Philolaus himself as a philosopher and scientist. His Philolaus conforms to Burkert's general notion of Pythagoreanism, according to which neither Pythagoras nor the early Pythagoreans had anything in common with philosophy and science. As C. Huffman noted later,

> Burkert's Philolaus is not responding to problems raised in the Presocratic tradition; he is not a natural philosopher. Rather he is attempting to translate an essentially religious view of the world inherited from Pythagoras into the foreign language of Ionian *physiologia*[3].

It is only natural, therefore, that in his own book about Philolaus Huffman, although drawing largely on Burkert's work, tried to overcome the limits of the latter's approach and to reconstruct Philolaus' philosophical and scientific teaching contained in the genuine fragments and testimonies[4].

I have to admit at once that the major part of Huffman's general as well as his particular conclusions about Philolaus' teaching seems to me very convincing; his book is undoubtedly

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* These notes were initiated by the discussions at the conference on Philolaus (Lille, November 1997), centred around Carl Huffman's exposé "The Status of Scholarship on Philolaus". I am very thankful to André Laks for his kind invitation to Lille, as well as to Christian Wildberg (Princeton University) and Brian Golding (Institute for Advanced Study, Princeton), who corrected my English and made some valuable suggestions. In quoting the Greek authors I used the standard English translations if available.

[1] An interesting paper by Ch. Riedweg, "Pythagoras hinterliess keine einzige Schrift" – ein Irrtum? // *Mus. Helv.* 54 (1997) 65–92, makes clear that the tradition of the absence of any of Pythagoras' books appeared not earlier than during the Hellenistic period. It does not follow from this, however, that in reality Pythagoras *did* have some writings.

[2] W. Burkert. *Lore and Science in Ancient Pythagoreanism* (Cambridge, Mass. 1972).

[3] C. Huffman. The Status of Scholarship on Philolaus, 2.

[4] C. Huffman. *Philolaus of Croton: Pythagorean and Presocratic* (Cambridge, Mass. 1993).

an exemplary study devoted to an individual Presocratic thinker. If some of my further notes mostly (though not exclusively) concern the issues that evoke doubt and criticism, it is only because from the perspective of my own studies of Pythagoreanism Philolaus appears to *continue* the philosophical and scientific tradition originated by Pythagoras[5], whereas Huffman in trying to "save" Philolaus considered it possible to sacrifice for this both Pythagoras and practically all Pythagoreans before Philolaus. Indeed, Huffman's Philolaus is not merely a "Pythagorean and Presocratic", he is in a sense the *first* Pythagorean, since before him no Pythagorean took up science and philosophy, and even if somebody did, he did not set forth his ideas in writing. Thus, Philolaus' fragments

> are not only our earliest primary text for Pythagoreanism but also… the only direct evidence for the first 150 years of Pythagoreanism at all[6].

Paraphrasing Huffman's words that Philolaus' fragments "remained to some extent hostage to Burkert's overall thesis about Pythagoras and Pythagoreanism", one can say that Huffman himself became a hostage to Burkert's overall thesis about the purely religious character of early Pythagoreanism. Following this thesis he was forced not only to ascribe to Philolaus the philosophical ideas of Pythagoras (e.g. the famous opposition πέρας–ἄπειρον) – for Pythagoras could not have any philosophical ideas – but also to isolate Philolaus from the Pythagorean science of the late sixth and the first part of the fifth century.

In "cutting" Philolaus off from his Pythagorean roots, Huffman thereby seriously exaggerates his originality. For example, according to Huffman, Philolaus' astronomical system appears not as a result of a modification of the preceding Pythagorean geocentric model but comes, as it were, out of nothing. The same can be said about Philolaus' mathematics, harmonics or medicine: the early Pythagorean tradition appears in the examination of his ideas only in those rare cases when it is really impossible to manage without it[7]. But even in these cases Huffman in no way tries to reveal the real connection of Philolaus' teaching with the ideas of his Pythagorean predecessors, such as Hippasus, Alcmaeon or Hippon.

It seems only natural that Huffman has serious problems in identifying the philosopher and scientist Philolaus as a Pythagorean. Indeed, if early Pythagoreanism consisted only of religious lore and mystical speculations about number, then was Philolaus a Pythagorean at all? Huffman solves this problem in the following way: his Philolaus, although not entirely ceasing to be a Pythagorean, becomes more and more of a "Presocratic":

> Philolaus should be regarded not just as representative of a narrow Pythagorean tradition but instead as also an important Presocratic thinker… In fact, Philolaus philosophy is so much the product of the Presocratic tradition that it is reasonable to ask in what sense we should call him a Pythagorean. Should he not rather be regarded as an independent Presocratic thinker, although under some Pythagorean influences, in the mold of Empedocles[8]?

In such an interpretation one cannot help noticing a certain paradox. On the one hand, Philolaus is the first and the main source for the first 150 years of Pythagoreanism (and not only for

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[5] L. Zhmud. *Wissenschaft, Philosophie und Religion im frühen Pythagoreismus* (Berlin 1997). Original Russian version: St. Petersburg 1994.
[6] Huffman. Status, 3.
[7] See, e.g. on Hippasus: Huffman. *Philolaus*, 148, 168 f.
[8] Huffman. Status, 6 (italics are mine).

us but for Aristotle as well)[9], and on the other hand he is not longer a 'real' Pythagorean. This paradox arises from Huffman's efforts to follow both the ancient tradition about orality and even secrecy of the Pythagorean teaching before Philolaus and Burkert's ideas, according to which early Pythagoreanism has nothing in common with rational (natural) philosophy and exact science, which, however, is precisely what Huffman finds in Philolaus' fragments.

I believe that such an approach can hardly lead to an adequate picture either of the historical background of Philolaus' teaching or of his own personality. It is the more regrettable, since Huffman's position is not necessarily connected with his analysis of Philolaus' fragments; it is rather imposed upon him from the outside, and that is why he could have easily abandoned it. Had Huffman accepted the evidence that before Philolaus in the Pythagorean school there was a hundred-year tradition of philosophical and scientific studies, he would be forced to sacrifice only a part of his hero's originality. In that case many other things which are hardly explicable from his present position would receive a much more adequate explanation.

As I tried to show in my book, there are absolutely no grounds to believe that Philolaus' book was the first written formulation of the Pythagorean doctrines[10]. Rather it was one of the many writings in which the Pythagoreans of the late sixth and the fifth centuries studied the philosophical and scientific problems they were interested in. Considered from such a perspective, Philolaus' work ceases to be the main source on Pythagoreanism both for us and for Aristotle. To be sure, among the Pythagorean theories referred to by Aristotle, some actually come from Philolaus. It is possible to verify this by comparing them with his preserved fragments. This does not mean, however, that everything or nearly everything that Aristotle says about certain anonymous Pythagoreans derives from Philolaus[11]. On the contrary, most of the Pythagorean doctrines which Aristotle describes do not find any correspondence in Philolaus' fragments, and we have no reason to read them into his book. Some of them, e.g. the theory of the "breathing cosmos" or the astronomical theory of the "celestial harmony" belong to a period much earlier than Philolaus. Some other, such as the famous "number philosophy", to which Aristotle returns again and again, is an artificial construction, which has a very remote, if any, relationship to the actual theories of the Pythagorean philosophers.

Since it is clear that not everything that Aristotle calls 'Pythagorean' really derives from this school[12], the more cautious one has to be when one tries to identify specifically Philolaic ideas in the Aristotelian material. Indeed, Aristotle never mentions Philolaus in scientific or philosophical contexts; he prefers to talk about the Pythagoreans or the so-called Pythagoreans. Aristotle calls Philolaus by name only once (*EE* 1225 a 30), referring to some gnomic dictum that does not necessarily come from Philolaus' book. Whatever this reservation means, it hardly supports the idea that Philolaus was "Aristotle's primary source of information on the Pythagoreans." It is noteworthy that a similar very reserved attitude of Aristotle towards Py-

[9] "Burkert and I agree that Philolaus' book is likely to be Aristotle's primary source of information on the Pythagoreans" (Huffman. Status, 8).
[10] Zhmud. *Wissenschaft*, 75 ff., 85 ff., 89 f.
[11] Cf.: J.A. Philip. Aristotle's sources for Pythagorean doctrine // *Phoenix* 17 (1963) 251–265. Incidentally, Philip thought that "of the Philolaus fragments as we know them, Aristotle would appear to know nothing" (255).
[12] See below, p. 17 f.

thagoras[13] is usually explained in the sense that the latter simply was not known to him as a philosopher and scientist or, at least, that he did not know what belongs to Pythagoras and what to his later followers. Why then, in exposing Philolaus' astronomical system, does Aristotle never say that it is Philolaus' and not merely a "Pythagorean" system? Why, in discussing the doctrines of Alcmaeon, Hippasus, Hippon, Philolaus, Archytas or Eurytus, does he never call them the Pythagoreans? Is it because they were not? Who then was? The problem of how Aristotle deals with Pythagoreanism and its individual representatives is evidently too complex to be resolved by supposing that he had no other direct sources on the early Pythagorean philosophy except Philolaus' book, and therefore that there was no Pythagorean philosophy before the last third of the fifth century.

Losing its privileged status as the earliest and – what is more – the only source on early Pythagoreanism, Philolaus' book acquires instead a solid basis in the tradition of the Pythagorean school, outside of which it cannot be adequately understood. The fact that the spheres of interests, e.g. of Alcmaeon, Hippasus and Philolaus coincided only partially should not be regarded as a certain deviation from the norm; the same can be observed with Parmenides and Zeno, and later with the students of Plato and Aristotle. In spite of all the variety of interests inside the Pythagorean school and the predisposition of its individual members toward different sciences, the problems that attracted the attention of Philolaus' predecessors, including Pythagoras himself, find a direct response in his writing. In philosophy Philolaus seems to be even closer to Pythagoras than some of his predecessors, who were less interested in the problems of cosmogony and ontology. His closeness to Alcmaeon and Hippon is clear from their common interest in medicine and physiology, as well as from the coincidence of their views on many particular issues in natural sciences[14]. In the field of harmonics Philolaus doubtlessly drew on the investigations of Pythagoras and Hippasus, and his astronomical and mathematical ideas also bear a distinctive trace of the early Pythagorean science.

It clearly follows from Philolaus' fragments that he was very familiar with all the four sciences of the Pythagorean quadrivium: geometry, arithmetic, astronomy and harmonics (cf. 44 A 7a). The same four μαθήματα were taken up by his contemporary Pythagorean Theodorus of Cyrene (43 A 2–4) and by his younger contemporary Pythagorean Archytas (47 A 14–19, B 1–2). But in contrast to Theodorus and Archytas, Philolaus did not prove to be a gifted mathematician and did not contribute anything to harmonics; the only discipline of the quadrivium in which he achieved something was astronomy. If we combine these facts with a tradition according to which Pythagoras took up all the four μαθήματα[15], and Hippasus at least three of them (geometry, arithmetic and harmonics, 18 A 4, 12–15), then we come to the conclusion that as a young man (i.e. before his escape to Thebes c. 450 BC) Philolaus received a "standard" Pythagorean education within the framework of the mathematical quadrivium. Having spent most of his life in Thebes, i.e. at the periphery of Greek culture of that time, with

[13] At least in Aristotle's extant treatises and with some notable exceptions (*Met*. 986 a 30; *Rhet*. 1398 b 14); the fragments of his lost works (fr. 152, 171 Gigon; *Protr*. fr. 18, 20 Düring) reveal a more positive attitude.

[14] See below, p. 9. It is significant that Hippon does not appear at all in Huffman's book: as a Pythagorean thinker, who lived before Philolaus and wrote two books (38 A 11), he is too inconvenient a witness. Cf. also *Philolaus*, 15 n. 25, where the discussion of Hippon is mysteriously omitted.

[15] Zhmud. *Wissenschaft,* 156 f., 168 f., 191 f., 211 f.

whom would he have studied mathematics there, all the more since he hardly felt an inner vocation to this science[16]?

Therefore, we may safely accept as attested that mathematics and harmonics, which were learned by Philolaus in his youth and later played such an important role in the formation of his philosophical system, and in particular of his epistemology, were not merely "Presocratic" sciences but precisely early Pythagorean. And although it is incorrect to say that "Philolaus is the first thinker self-consciously and thematically to employ mathematical ideas to solve philosophical problems"[17], because such a thinker was Parmenides, and after him Zeno, it is revealing that the Eleatics also derived their knowledge from the very same source, namely from early Pythagorean mathematics[18].

"Neither Aristotle… nor Alexander… knows of any other Pythagorean astronomical system than that of Philolaus," suggests Huffman[19]. In fact, both of them know of such a system[20], but even if they did not, this would not be of crucial importance. What is important, is that Philolaus' system, as we know it, is too advanced to emerge simply from nothing. The spherical shape of the earth, the correct order of the five planets, their circular movement in the direction opposite to that of the fixed stars, and many other features characterise a developed astronomical theory. None of these astronomical ideas was considered in the ancient tradition as Philolaus' discovery. To whom then belongs the system that Philolaus modified by postulating a central fire and a counter-earth[21]? To the "Presocratics"? But outside the Pythagorean school we do not know of any Presocratic thinker whose astronomy included all of the features listed above[22].

On the other hand, the evidence ascribes to Pythagoras the idea of the sphericity of the earth (D.L. VIII,48) and the division of the earth into five zones (*Dox*. p. 378, 340) as well as the identification of the Morning and the Evening star with the planet Venus (Aristox. fr. 24 Wehrli). The doxographic tradition connects Alcmaeon with the notion of the independent movement of the planets in the direction opposite to that of the fixed stars (24 A 4), while Aristotle's student Eudemus, the best expert in the history of astronomy, testifies that the Pythagoreans were the first to introduce the order of the celestial bodies accepted in his own time (fr. 146 Wehrli), i.e. moon, sun, Venus, Mercury, Mars, Jupiter, Saturn. Moreover, the early

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[16] It is not by chance that we do not know of his teachers. It seems that he did not have any in Thebes.
[17] Huffman. Status, 5.
[18] On the origin of deductive proof, used in the Eleatic philosophy, from the Pythagorean mathematics see: A. Zaicev. *Das griechische Wunder* (Konstanz 1993) 172 f.
[19] Huffman. *Philolaus*, 281.
[20] On Alexander (*In Met. comm*. p. 39.24–40.9 = fr. 162 Gigon) see: Zhmud. *Wissenschaft*, 221 f. Nothing in Aristotle's description of the "celestial harmony" at *De caelo* 290 b 12–29 suggests that he had in mind Philolaus' system. Besides, he mentions in the astronomical context not only Alcmaeon (*De an*. 405 a 29 = 24 A 12) but also Pythagoras (*Protr*. fr. 20 Düring).
[21] The difference between the two systems is clear from the fact that in Philolaus' astronomy the heavenly bodies which are nearer to the centre revolve faster (the earth in 24 hours, the moon in 28 days, etc.), while in the early Pythagorean system the arrangement was just the opposite: the more distant from the centre the faster is the movement. In addition to this, in Philolaus' system there is no trace of the very idea of the "celestial harmony" (as Huffman has to admit, *Philolaus*, 282), since the movement of the earth would destroy the whole cosmic music. To be sure, Huffman does not consider Philolaus an originator of the doctrine of the "celestial harmony", but who then originated this doctrine and, what is more important, the astronomical system corresponding to it, if not the early Pythagoreans?
[22] On Parmenidas' astronomy see: Zhmud. *Wissenschaft*, 211 ff.

Pythagoreans put forward a basic assumption of the whole Greek astronomy, namely that the movements of the sun, the moon and the five planets are circular and uniform[23], an assumption that Philolaus tacitly accepts. It seems impossible to ignore an obvious fact that all these ideas, ascribed by the tradition either to Pythagoras or to individual Pythagoreans or to the Pythagorean school as a whole are incorporated in Philolaus' astronomical theory, while nothing indicates that he was their author. Thus, one has to choose between imputing to Philolaus an extraordinary originality as an astronomer (an originality, moreover, which went unnoticed by both Aristotle and the rest of Greek tradition), and admitting that after all there was a Pythagorean astronomy before Philolaus (as much as there were Pythagorean mathematics, harmonics or philosophy). I do not consider this choice to be very difficult.

2. Time

The early Pythagorean cosmogony describes the origin of the world as the process of inhaling πνεῦμα from ἄπειρον κενόν that surrounds the cosmos. As a good example of the sixth century natural philosophy, having distinct parallels in the cosmological theories of Anaximander (12 A 9–10, 23) and Anaximenes (13 A 5–7, B 2), the idea of the breathing universe clearly does not fit the picture of Philolaus' cosmos which arises from τὰ ἄπειρα καὶ τὰ περαίνοντα, united by cosmic ἁρμονία (44 B 1). The other pieces of evidence, e.g. Xenophanes' belief that the cosmos does not breathe (21 A 1.26)[24] and that the earth is not surrounded by air (21 A 32–33), or the identification of ἀήρ with κενόν by Alcmaeon (24 A 5) also point to the sixth – early fifth century way of thinking[25]. Therefore, the Pythagorean cosmogony as represented by Aristotle and his commentators belongs to a period much earlier than Philolaus. Huffman objects to this idea, but his arguments do not seem to me very persuasive. Indeed, assuming that Aristotle had no sources about early Pythagoreanism except Philolaus' book, Huffman is compelled to look for some connection of this cosmogony with Philolaus' philosophy, or at least to show that it is not after all so archaic as it seems to be:

> Some scholars have seen the concept of a breathing cosmos as archaic and suggested that this concept shows that breath and void had not yet been distinguished… However, [in Aristotle, fr. 166 Gigon][26] breath and void are listed along with time as three things breathed in… It seems much more probable that these are three distinct things that are breathed in and that breath and void are distinguished[27].

Huffman regrettably forgets to say that there is no word about χρόνος in three passages from the "Physics" and "Metaphysics" about the Pythagorean cosmogony, as well as in the explanations and paraphrases of Aristotle's commentators[28]. Thus, from the eight references to Py-

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[23] Gemin. *Eisag*. I, 19; cf. Alcmaeon (24 A 12).

[24] See, e.g.: J. Burnet. *Early Greek Philosophy* (London 1920) 108; M. Untersteiner. *Senofane. Testimonianze e frammenti* (Firenze 1955) 9.

[25] The identification of air and void was refuted by Empedocles (31 B 100) and Anaxagoras (59 A 68–69).

[26] ἐν δὲ τῷ Περὶ τῆς Πυθαγόρου φιλοσοφίας πρώτῳ γράφει τὸν μὲν οὐρανὸν εἶναι ἕνα, ἐπεισάγεσθαι δ' ἐκ τοῦ ἀπείρου χρόνον τε καὶ πνοιὴν καὶ τὸ κενόν, ὃ διορίζει ἑκάστων τὰς χώρας ἀεί.

[27] Huffman. Status, 17.

[28] Arist. *Met.* 1091 a 12–20, *Phys*. 203 a 1–13, 213 b 24–28; cf. *Dox*. p. 338.13–15. Philop. *In Phys. comm*. p. 615.21 f.; idem. *In De gen. anim. comm*. p. 107.14; Simpl. *In Phys. comm.* p. 651.26 f. Huff-

thagorean cosmogony only *one* mentions time, while all the others affirm (some of them *expressis verbis*) that the Pythagoreans *identified* air with the void[29]. This fact alone should alert those who try to make too far reaching conclusions from Aristotle's fr. 166, preserved in Stobaeus (*Dox*. p. 316.11 ff.). The idea of time being "breathed in" from the unlimited void that surrounds the cosmos always posed a problem for the interpreters[30]; the explanations of Zeller and Guthrie, who dealt with this problem at length, leave the impression that they themselves were not satisfied with their arguments[31]. The suspicion that χρόνος in this fragment is out of place becomes even stronger if we read an entry on the following page of Diels (*Dox*. p. 318.3), entitled Περὶ χρόνου: Πυθάγορας τὸν χρόνον τὴν σφαῖραν τοῦ περιέχοντος εἶναι. It seems very tempting to suppose that χρόνος was inserted by a doxographer into Aristotle's fragment from that very entry[32]. This would reasonably explain why Aristotle and his commentators were so conspicuously silent about time in the context of the Pythagorean cosmogony, as well as of Pythagorean philosophy in general. Actually, we do not know of any Pythagorean theory *of what time is*, as opposed to some gnomic dicta and the idea of the eternal recurrence (Eud. fr. 88, 90 Wehrli), and it would be anachronistic to project such a theory into the sixth – early fifth century[33]; even in the evidence on Philolaus and Archytas we do not find anything pointing in this direction. For the same reason the doxographic entry about Pythagoras' concept of time also looks very suspicious; to my knowledge it has no correspondence in the early Pythagorean tradition. How did this entry arise?

In the "Physics", Aristotle consecutively discusses the concepts of infinite, void, and time. In the first two cases he explicitly refers to the distinct Pythagorean theories on this account, but not in the last, where he remarks that his predecessors did not say much about what time is. Indeed, he quotes only two opinions: some say time is the movement of the universe and the other say it is the sphere itself (οἱ μὲν γὰρ τὴν τοῦ ὅλου κίνησιν εἶναί φασιν, οἱ δὲ τὴν σφαῖραν αὐτήν, 218 a 33), but does not specify who are these thinkers. The

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man briefly mentions these three Aristotle's references, without quoting them in full, whereas the entry in *Dox*. p. 338.13–15 and the commentaries of Philoponus and Simplicius remain unmentioned.

[29] See, e.g.: "the followers of Pythagoras say that there is a void outside of the cosmos, and the cosmos breathes out of it and in it" (*Dox*. p. 338.13–15); "they say that the void enters the cosmos as if it breathed in a sort of breath from that which lies outside" (Simpl. *In Phys. comm.* p. 651.26 f.).

[30] Kranz noted in the apparatus that χρόνος is "zweifelhaft" (*DK*, p. 460 not.), and Diels suggested here ἐκ τοῦ ἀπείρου χρόνου (cf. 68 A 39).

[31] E. Zeller. *Die Philosophie der Griechen.* 6. Aufl. (Leipzig 1919) Bd. I.1, 545 f., W.K.C. Guthrie. *A History of Greek Philosophy*. V. 1 (Oxford 1962) 280 f., 336 ff. In a short article devoted to this issue A. Petit although admits that Aristotle's fragment is our *only* evidence on extra-cosmic time in the whole Pythagorean doxography, tries to integrate it in his philosophical interpretation, without paying due attention to the source problems (L'espace vide et les temps illimité dans le pythagorisme ancien // *L'Espace et les Temps. Actes du XXIIe Congrès de l'Assoc. Soc. Phil. de Langue Française* [Dijon/ Paris 1991] 97–102).

[32] On Stobaeus' working method and the possible contaminations in this chapter see: K. Algra. 'Posidonius' Conception of the Extra-Cosmic Void: the Evidence and the Arguments, *Mnemosyne* 46 (1993) 473–505; J. Mansfeld, D. Runia. *Aëtiana. The Method and Intellectual Context of a Doxographer* (Leiden 1997) V. 1, 196 ff., 251 f.

[33] Neither in Aristotle's "Physics", nor in Theophrastus there are any traces of such a theory belonging to the early Presocratics.

first opinion belongs to Plato, as attested by Theophrastus and Eudemus[34], the second was later attributed to the Pythagoreans and so, I believe, found its way to the doxographic entry (σφαῖρα τοῦ ὅλου = σφαῖρα τοῦ περιέχοντος). In his commentary on this place Simplicius remarks:

> Some attribute this idea to the Pythagoreans, having probably wrongly understood the general definition by Archytas, who said that time is διάστημα τῆς τοῦ παντὸς φύσεως, and some to the Stoics (*In Phys. comm.* p. 700.19 f.).

As follows from the other passage of Simplicius' commentary (ibid. p. 786.11), "Archytas' definition" of time derives from Iamblichus, who in turn took it from the pseudo-Pythagorean "Categories", attributed to Archytas[35]. All this suggests, first, that it is hardly possible to prove that the second definition of time in the "Physics" belongs to the Pythagoreans; at least, neither Aristotle's students[36] nor his best commentators could find in Aristotle or in the authentic Pythagorean tradition anything in support of this attribution[37]. Therefore, "Pythagoras' definition" of time in Stobaeus is likely to be based on the wrong guess. Secondly, even if my explanation of how χρόνος got into Aristotle's fr. 166 is wrong, it is highly questionable that this fragment originally contained anything on breathing in time: though the fragment comes from the treatise "On the Pythagoreans", certainly known to Alexander and Simplicius, they do not mention this idea, as, in fact, do none of the other ancient commentators of the "Physics"[38]. Thirdly, there is an obvious difference between the notion of time as being the sphere of cosmos, – "for everything is both in time and *in* the sphere of the all", as the argument quoted by Aristotle runs (218 b 5), – and the idea that time exists *outside* of the cosmos and is breathed in together with the void. These notions are barely compatible with one another, especially if we consider them in context, i.e. as a part of a wider world picture and not just as some formal

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[34] "Some say that time is "the motion of the whole" and its rotation, as Eudemus and Theophrastus and Alexander think Plato did" (Simpl. *In Phys. comm.* p. 700.16 f. = Eud. fr. 82a Wehrli = Theophr. *Phys. op*. fr. 15). See: Pl. *Tim*. 39c–d.

[35] H. Thesleff. *The Pythagorean Texts of the Hellenistic Period* (Åbo 1965) 24.15.

[36] It is important to bear in mind that Simplicius extensively used Eudemus' "Physics" that closely followed Aristotle's "Physics", amplifying it with the doxographical references. He preserved no less than ten Eudemus' fragments, devoted to the notion of time (fr. 82–91 Wehrli), of which fr. 82 is about Plato's definition of time, fr. 88 criticizes the Pythagorean theory of the eternal recurrence, and fr. 90 mentions some unnamed Pythagorean, who called time ἀμαθέστατον (cf. Arist. *Phys*. 222 b 18). It seems that Eudemus had nothing more to say on the Pythagorean notion of time. Cf. Wehrli's commentary to Eud. fr. 82.

[37] Burkert (76) brings not philosophical but rather poetical parallels, none of which comes from the Pythagorean tradition. It seems that the early Pythagoreans were interested not so much in the abstract notion of time, as rather in καιρός as "critical time" or "opportune time", which they connected with the number seven (cf. 38 A 16). As is clear from Kranz' *Wortindex* (*DK* III, 223), they shared this interest with many thinkers of the archaic and classical periods.

[38] At *De Caelo* 279 a 11 f. Aristotle, continuing his polemic against the existence of the unlimited (*Phys*. 203 b 16 ff.), denies that there is place, void or time outside Heaven. Commenting on this passage, Simplicius does not mention the Pythagoreans (*In De Caelo comm*. p. 284.5 ff.), he understands it as a logical sequence of what was proved earlier, and not as an argument against any specific theory. According to Aristotle's theory there is no time without movement (*Phys*. 223 a 20–21) and no movement without place. It seems quite logical that further (*De Caelo* 279 a 18–b3) Aristotle tries to prove the eternity of the divine Unmoved Mover, existing outside the cosmos. Cf.: "Aristotle is careful to restrict time to the sphere of motion and therefore to that of natural objects capable of motion and so of coming into being and passing away. Items that exist eternally are not in time", J. Annas. Aristotle, Number and Time, *Philos. Quarterly* 25 (1975) 101.

"definitions" of time. Of course, given some dialectical ingenuity, it is always possible to make them coherent (although Zeller refuses to do this)[39], e.g. to take them as representing different stages of the cosmological process or different stages of the Pythagorean philosophy. To be sure, the concept of time being inhaled from outside of the cosmos looks more archaic than its definition as σφαῖρα τοῦ ὅλου; the problem, however, is that the later Pythagoreans, e.g. Archytas, continued to believe that there is an infinite outside of the sphere of the fixed stars (Eud. fr. 65 Wehrli = 47 A 24), so why would they insist that πάντα ἐστὶ ἐν τῇ τοῦ ὅλου σφαῖρα?

In Philolaus' views one can also find a parallel, though a remote one, to the early Pythagorean cosmogony. He states that after birth the animal immediately draws in the cold breath outside and then breathes it out; in this way the outer breath cools the body down (44 A 27). Although the *image* really resembles the process of the cosmic breathing, it should be noted that the *idea* Philolaus tries to express here has hardly anything to do with his own cosmogony. He rather echoes (and at some point develops) Hippon's concept (38 A 10) that the soul makes the body cooler by breathing, since the soul is cold by its nature (Hippon derived ψυχή from ψυχρόν)[40]. It is not by chance that Philolaus applies the same kind of argument as Hippon, including the use of etymology (he derived φλέγμα from φλέγειν, to burn).

To sum up the situation, we may say that Philolaus' cosmos originates not from the ἄπειρον κενόν, which is inhaled by the cosmos from outside and is limited by πέρας (cf. *Met*. 1091 a 15 f.), as in the early Pythagorean cosmogony, but from much more abstract principles, τὰ ἄπειρα καὶ τὰ περαίνοντα (which cannot be identified with any physical substance or element), united by cosmic ἁρμονία (44 B 1). Philolaus repeatedly stresses the role of ἁρμονία uniting the opposite principles (44 B 6); the first thing that is fitted together out of them is Hestia, located in the middle of his geometrically structured cosmos (44 B 7, 17). These fundamental features of Philolaus' *cosmology* distinguish it from the archaic physical *cosmogony* described by Aristotle and his commentators. If Hestia has any counterpart in the earlier Pythagorean tradition, then rather in Hippasus' fire (18 A 7).

On the other hand, apart from these and many other differences, there is also an obvious correspondence between concepts and principles of Philolaus' cosmology and the views of Pythagoras, Hippasus, Hippon and the other Pythagoreans. Without going into further details, one can say that this is only one of the many issues, where it is possible to show both an affinity of Philolaus to the Pythagorean predecessors and an individuality of his thought. Such an approach, it seems, neither undermines Philolaus' originality nor ignores the influence on him of non-Pythagorean, especially Eleatic philosophy. In fact, this is in perfect accord with the general thesis of Huffman's book on Philolaus as a Pythagorean and a Presocratic.

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[39] Cf. an attempt of H. Cherniss (*Aristotle's Criticism of Presocratic Philosophy* [Baltimore 1935] 214 ff.) that seems to me unsuccessful.

[40] Cf. the similar considerations in Diogenes of Apollonia (64 A 28) and in the Hippocratic treatise *De natura pueri*, c. 12: while an embryo exhales the warm breath, a mother inhales the cold air.

3. Numbers, especially the number ten

In two articles, published in "Phronesis" almost simultaneously[41], and in two subsequent books, Huffman and I tried to challenge the view traditional since Aristotle that number was the main principle of Pythagorean philosophy. Huffman's starting point was a clear incongruity between Aristotle's description of the Pythagorean "number philosophy" and what was said on number in Philolaus' fragments. Indeed, contrary to the assertions repeatedly made by Aristotle, the cosmos of Philolaus has arisen and consists not of numbers or corporeal units, but of things unlimited and limiting (44 B 1–2). It is these two kinds of things that Philolaus calls φύσις and ἀρχή of all (44 B 1, 6); he has no other beginnings. And number in Philolaus appears in an epistemological but not an ontological context (44 B 4–5).

My starting point was the same discrepancy between what is known about ἀρχαί of the early Pythagoreans and anonymous "number philosophy" that Aristotle ascribes to this school without mentioning even once to whom it actually belongs. It is stated in the "Metaphysics" that none of the Pythagoreans had said anything about sensible principles (989 b 30ff, 990 a 17), while Hippasus' ἀρχή was fire (*Met*. 984 a 7) and Hippon's water (*Met*. 984 a 4). According to Aristotle, Pythagoreans gave explanations of all things by means of quantitative characteristics, yet in Alcmaeon and Hippon we find only qualitative opposites, principally hot and cold (*Met*. 986 a 27; 32 A 5). Thus, Aristotle himself confirms once more that the sensible, corporeal ἀρχαί and the qualities connected with them were at the basis of natural philosophy of the early Pythagoreans known to him – and in this way they did not differ from other Presocratics. Strange as it may seem, they say nothing about number as the substance of the world.

Judging by the surviving evidence, Philolaus was the first of the Pythagoreans to have placed number in a philosophical context. Why then was this context specifically epistemological, and not ontological? Has Philolaus made any clear distinction between ontology and epistemology? And was the "number theory" of the Pythagoreans not at the same time their cosmology, as Aristotle testifies? H. Schibli poses all these reasonable questions in his recent article[42], being clearly dissatisfied with the "radical revision" of fifth-century Pythagoreanism undertaken by the two "detractors of number". His own answer amounts to the assertion "that the Pythagoreans identified physical substances and numbers in the way Aristotle says they did"[43], and more specifically that Philolaus' central fire is at the same time the arithmetical unit, 1.

I have quite different answers to the questions posed by Schibli but first I would like to say that the "radical revision" was undertaken by both "detractors of number" with one principal aim: to bring "our understanding, imperfect as it may be, of fifth-century Pythagoreanism" closer to the Pythagorean texts of this period. If such a text contains something that contradicts Aristotle's understanding of the Pythagorean ideas, it is both methodologically and logically preferable to base our judgement on the primary text, while taking into account what Aristotle says on this issue. This is an old and simple rule, hardly questioned by anyone in the

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[41] C. Huffman. The Role of Number in Philolaus' Philosophy // *Phronesis* 33 (1988) 1–30; L. Zhmud. "All is Number"? "Basic Doctrine" of Pythagoreanism Reconsidered // *Phronesis* 34 (1989) 270–292.
[42] H.S. Schibli. On "The One" in Philolaus, Fragment 7 // *CQ* 46 (1996) 114–130.
[43] Ibid., 127.

history of Presocratic philosophy, and it has nothing in common with "professional" accusations that Aristotle was fundamentally unable to understand the Presocratics, which form the point of departure in some recent books[44].

Second, unlike Huffman, for whom Philolaus' book is the only Pythagorean text of the fifth century, I am not a partisan of the *Einquellentheorie*. Both Aristotle's surviving writings and the fragments of his lost treatises unequivocally demonstrate that he knew of theories and books of the Pythagoreans before Philolaus. And, whereas Huffman did not find in Philolaus' fragments any identifications of things and numbers (although there is something here, what can be reasonably called "philosophy of number"), my search for anything like this in the earlier Pythagorean tradition brought even less. The authentic fragments and testimonies, coming from the individual Pythagoreans of that time contain no "philosophy of number", indeed, no mention of number in a philosophical context[45].

To be sure, Pythagoras and his followers did take up mathematics, astronomy and harmonics, in which number(s) played a prominent role, and they undoubtedly possessed an understanding of what *mathematical* number is. Needless to say, however, mathematics and mathematical definition of number appear much earlier than *philosophy* of mathematics and philosophical understanding of number. Here, I believe, is an answer to Schibli's first question. The epistemological theory of number developed by Philolaus is rooted not in an earlier "number theory" or "number cosmology", previous to him, but in Pythagorean *mathematics*, including mathematical astronomy and harmonics[46]. By the time of Philolaus the exact sciences had repeatedly proved their ability to be a powerful instrument of cognition, in fact, they became a real paradigm of that kind of cognition which can provide an irrefutable truth. Therefore, it is very plausible that behind Philolaus' words "And indeed all things that are known have number" (44 B 4) stands the firm belief of so many mathematicians that things are cognizable to the extent that they can be expressed in numbers.

A professional geometer at the end of the fifth century would perhaps generalise Philolaus' formula by saying "… have number or *magnitude*", – but Philolaus was not a professional geometer. He was a philosopher, who tried to combine the traditional Pythagorean theory of two opposite qualitative principles (in his case τὰ ἄπειρα and τὰ περαίνοντα) with a new philosophical understanding of the way or method we get to know the world. Is it not quite natural that his attempts to harmonise these two sets of concepts were of limited success and that some of his ideas were defined not clearly enough and allowed different interpretations? (We should not also forget about very fragmentary character of our sources.) I am not sure at all that Philolaus would have been able to explain clearly, what exactly the distinction between the ontological and epistemological levels of his theory is. But again, to distinguish does not mean to be able to explain the distinction. These two levels are different because Philolaus does not confuse them: his cosmos consists just of unlimited and limiting things, fitted together by ἁρμονία, and not of numbers.

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[44] A. Capizzi. *The Cosmic Republic: Notes for a Non-Peripatetic History of the Birth of Philosophy in Greece* (Amsterdam 1990); P. Kingsley. *Ancient Philosophy, Mystery, and Magic: Empedocles and the Pythagorean Tradition* (Oxford 1995).

[45] The so-called acousmatic tradition deserves a special treatment, see: Zhmud. *Wissenschaft*, 75 f.

[46] Not by chance he has stressed the importance of λόγος that comes from τὰ μαθήματα (44 A 29). See Huffman's commentary: *Philolaus*, 199 f.

While some Presocratics did believe that "like is cognizable by like", the progress of mathematics and its application to the physical world was not based on this primitive isomorphism. From the time of Anaximander and Pythagoras onwards the exact sciences revealed the geometrical structure of the cosmos and numerical structure of musical harmony without maintaining that the heavenly bodies or harmonious intervals *consist* of numbers. For those interested in numerical proportions of the sides of the triangle, in distances between the planets or in intervals of the musical scale, all these things have to be countable and measurable. Putting this in a philosophical language of that time, they must "have number", but why should they be *made* of arithmetical units?

Indeed, already in the early fifth century it has been deductively proved by Hippasus that the diagonal of the square, being incommensurable with its sides, *cannot* be expressed either by a whole or by a fractional number. This theory was developed further by Theodorus of Cyrene who found a whole class of irrational magnitudes, which nevertheless could be readily constructed geometrically. Later, his student Theaetetus elaborated a general theory of the irrational magnitudes. Did all this have any significance for Philolaus' epistemological thesis that all that is known has to have number? Or rather did philosophical evaluation of these mathematical discoveries come much later, as is usually the case? Judging by what we know of the theories of Philolaus' followers, Eurytus and Ecphantus, they were not at all embarrassed by the problem of irrationality: Eurytus tried to "determine" the number of a man or of a horse (45 A 2–3), Ecphantus came to the idea of corporeal monads (51 A 2). This implies that Philolaus too was hardly striving to bring his epistemology into conformity with the latest stage of mathematics.

Schibli is right in that Aristotle really understood (or better to say, misunderstood) Pythagorean cosmology as their "number theory". But Philolaus' fragments, as Huffman convincingly shows, do not support such an understanding[47], and this view should be taken for what it really is: an Aristotelian construction, where the ideas of some later Pythagoreans (including Philolaus) are combined with various interpretations of Pythagoreanism, offered in the Platonic Academy[48]. How Aristotle builds from this material the "Pythagorean number philosophy" can be shown by one conspicuous example. While discussing Pythagorean philosophy, Aristotle notes:

> Since the number ten is considered to be perfect and to comprise the whole nature of numbers, they also assert that the bodies which revolve in the heavens are ten; and there being only nine that are visible, they make the counter-earth the tenth (*Met*. 986 a 10).

Meanwhile, from his earlier works, "On the Heaven" (293 b 21) and "On the Pythagoreans" (*Dox*. p. 360.1 ff = fr. 170 Gigon), to which he himself refers in the "Metaphysics"[49], it follows that the counter-earth was introduced in order to explain why lunar eclipses are more frequent than those of the sun; the same explanation is mentioned by Philip of Opus (58 B 36), who was a professional astronomer. Notwithstanding, whether this explanation is right or

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[47] Huffman. *Philolaus*, 57 ff.
[48] Zhmud. *Wissenschaft,* 261 ff.
[49] διώρισται δὲ περὶ τούτων ἐν ἑτέροις ἡμῖν ἀκριβέστερον. Cf. Alex. *In Met. comm*. p. 41.1: λέγει δὲ περὶ τούτων καὶ ἐν τοῖς Περὶ οὐρανοῦ καὶ ἐν ταῖς τῶν Πυθαγορικῶν δόξαις ἀκριβέστερον.

not[50], it contradicts the first one, and one can readily surmise that either Aristotle was not satisfied with this explanation or that he was not sure exactly why Philolaus introduced the counter-earth and interpreted it in the same work at one time astronomically, at another arithmologically[51].

If Philolaus was so fixed on the number ten that he decided to think up a tenth celestial body, he did not need to introduce the counter-earth at all, since together with the central fire (Hestia) there were already 10 celestial bodies in his system: five planets, earth, moon, sun, the sphere of the fixed stars and Hestia. Thus, the counter-earth would be the eleventh heavenly body! To be sure, Aristotle speaks about 10 *moving* bodies, and since Hestia does not move, he leaves it out. But if Philolaus was really motivated primarily by arithmological considerations, what could prevent him from counting Hestia as well[52]? In any case, Aristotle himself, with his characteristic inconsistency in all Pythagorean matters, elsewhere assigns to Hestia the first place in the cosmos: the Pythagoreans said that

> there is a particular number proper to each place in the universe. For at the center is 1 (for the center is the first place in the universe); after the center is 2… and in this way the number of things being constituted becomes greater as they keep moving from the center (Alex. *In Met*. p. 74.9 ff. = fr. 163 Gigon)[53].

Now, if Hestia is on the first place, the sphere of the fixed stars must be on the eleventh!

What ground do we have for thinking that the Pythagoreans did attach any significance to the number ten, except for Aristotle's commentary on the counter-earth? The famous table of the *ten* opposites (*Met*. 996 a 22 ff.), which he ascribes to some specific group of Pythagoreans, is now generally considered to be a product of later systematisation under obvious Platonic influence[54]. The acousmatic tradition is silent about the number ten, and the fragments of Philolaus (44 A 11–13, B 11) and of Archytas (47 B 5) on the decad are unanimously regarded as pseudo-Pythagorean. Hippon's "medical arithmetic" is based on the traditional number seven that he called the "most powerful"[55], whereas Eurytus was interested in numbers that were much greater than ten (45 A 2–3). Plato, who borrowed so much from Pythagorean mathematics, never calls ten the "perfect number"; although the expression τέλειος ἀριθμός occurs twice in his dialogues, it refers in one case to the so-called "nuptial number" (*Res*. 546 b 4), and in the other to the "Great Year" (*Tim*. 39 d 3-4). This suggests that *before* Plato there was hardly any doctrine on the perfection of the number ten, at any rate, we have no evidence

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[50] Cf. Burkert, 344; Huffman. *Philolaus*, 246 f.

[51] Alexander and Simplicius, referring to the book "On the Pythagoreans", connect the perfect number ten with the counter-earth (Alex. *In Met. comm*. p. 40.27 = fr. 162 Gigon; Simpl. *In De Caelo comm*. p. 512.4 ff. = fr. 169 Gigon). To give different and often incompatible explanations to the same idea is quite characteristic for Aristotle's interpretation of Pythagoreanism (Zhmud. *Wissenschaft*, 273 f.).

[52] After all, nothing has hindered Speusippus to believe that an equilateral triangle has only one side and one angle, – just because he was interested in arithmology, and not in mathematics or astronomy. See below, p. 16.

[53] Tr. Huffman. Alexander refers here to the second book of Aristotle's treatise "On the Pythagoreans".

[54] See below, p. 17.

[55] 38 A 16; cf. Solon, fr. 23 Gentili–Prato. Numbers 3, 4 and 7 play a prominent role in the passage of the Hippocratic treatise *On Seven-Month Babies* (c. 9), where Pythagorean influence is quite possible (Burkert, 262 f.). What number is meant here under τέλειος ἀριθμός, remains unclear, but definitively not the number ten.

to their effect. But Plato, with his predilection for various "significant" numbers and for arithmology in general, could pave the way for such a doctrine or even play some role in its formation.

It is worth noting that Aristotle does not speak about the Pythagorean *origin* of this doctrine; rather, he refers to the already existent theory that is supported also by the Pythagoreans: ἐπειδὴ τέλειον ἡ δεκὰς εἶναι δοκεῖ…, καὶ τὰ φερόμενα κατὰ τὸν οὐρανὸν δέκα μὲν εἶναί φασιν (note δοκεῖ, but φασίν). Taken together with his other references to this theory in the "Metaphysics", where he obviously has the Platonists in mind[56], this testimony confirms that Aristotle knew of it not from the Pythagorean but from the Platonic sources. One of the most significant among them seems to be Speusippus' treatise "On the Pythagorean numbers": half of it was devoted exclusively to the number ten (fr. 28 Tarán = 44 A 13), and Aristotle's remark looks very close to what Speusippus says on this account[57]. Speusippus was not, of course, the only Platonist interested in the doctrine on the decad. As a repeated criticism of this doctrine in the "Metaphysics" shows, it was significant for the whole Academy, and in the "Physics" Aristotle connects it with Plato himself[58]. For us, however, Speusippus is important as a most natural link between the Pythagoreans and the Platonists (at least, for this specific theory), all the more as his influence is discernible also in the table of the ten opposites[59].

What, indeed, could prevent Aristotle from taking some ideas of the treatise entitled *On the Pythagorean numbers* for an authentic doctrine of the Pythagoreans[60], especially that of Philolaus, whose name seems to be mentioned by Speusippus[61]? One of the most important Aristotle's sources for early Pythagoreanism was a book by Anaximander of Milet (the younger) Συμβόλων Πυθαγορείων ἐξήγεσις (58 C 6). Anaximander included in his collection of the Pythagorean σύμβολα very heterogeneous material, the substantial part of which had nothing in common with the Pythagoreans[62], and although Aristotle pointed out that the *explana-*

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[56] 1073 a 17–22, 1084 a 12 – b 2, 1088 b 10–11. See: E. Frank. *Plato und die sogenannten Pythagoreer* (Halle 1923) 257 n.1; W.D. Ross, ed. *Aristotle's Metaphysics*. V. II (Oxford 1958) 445 ff.; J. Annas. *Aristotle's Metaphysics. Books M and N* (Oxford 1976) comm. ad loc.

[57] Aristotle: ἐπειδὴ τέλειον ἡ δεκὰς εἶναι δοκεῖ καὶ πᾶσαν περιειληφέναι τὴν τῶν ἀριθμῶν φύσιν. Speusippus: ἔστι δὲ τὰ δέκα τέλειος, καὶ ὀρθῶς τε καὶ κατὰ φύσιν εἰς τοῦτον καταντῶμεν παντοίως ἀριθμοῦντες… ἔτι πάντες οἱ λόγοι ἐν τῶι ι', κτλ. See: Frank, 251 f., 256 f.; H. Flashar. *Aristoteles*. *Problemata Physica* (Berlin 1962) 567 f.

[58] 206 b 32. Aristotle probably means here his oral teaching, namely a lecture *On the Good*, since in the Platonic dialogues, as we have noted, τέλειος ἀριθμός does not refer to the number ten.

[59] Frank, 239 ff., Burkert, 51 f.; see below, p. 17.

[60] Nothing suggests that the title Περὶ Πυθαγορικῶν ἀριθμῶν was given to this work later, and not by Speusippus. We can compare it with the works of the other Academics: Πυθαγόρεια and Περὶ ἀριθμῶν by Xenocrates (D.L. IV, 13), Περὶ τῶν Πυθαγορείων by Heraclides of Pontus (fr. 22 Wehrli), Περὶ τῶν Πυθαγορείων by Aristotle (fr. 155–179 Gigon) and Περὶ πολυγώνων ἀριθμῶν by Philippus of Opus (20 T 1 Lasserre).

[61] *DK*, p. 400.24. On mentioning Philolaus see: L. Tarán. *Speusippus of Athens* (Leiden 1981) 260 f., cf. Burkert, 246; Huffman. *Philolaus*, 362. I do not see any reason why Speusippus could not mention Philolaus. His uninterrupted search for the new similarities and correspondences between numbers, figures and solids can be compared with the related, although fortunately not so developed interest of Philolaus: "Some, following Philolaus, believe that the harmonic mean is called so from its attendance on all geometric harmony, and they say geometric harmony is the cube… For in every cube there are 12 sides, 8 angles, and 6 surfaces. Then, 8 is the mean of 6 and 12 in accordance with the harmonic proportion" (44 A 24).

[62] Zhmud. *Wissenschaft*, 93 f., 97 ff.

tions given to some of the σύμβολα are late, he took the σύμβολα themselves as genuinely Pythagorean. The same could happen with the book on the Pythagorean numbers: even taking into account that the *explanation* of the perfection of the decad belongs to Speusippus himself, Aristotle could readily credit the Pythagoreans with an original idea, since he firmly believed that for them all is number. It is possible also that Speusippus himself tried to explain the counter-earth by the reference to the perfect number ten, – this would be a good reason for mentioning Philolaus. But even if Speusippus did not mention him, it was only natural for Aristotle to guess that the appearance of the tenth moving body in Philolaus' system is somehow connected with the arithmological doctrine on the decad, that was passed off by the Platonists as the Pythagorean. After all, the conviction that Speusippus has been inspired by the Pythagorean arithmology was shared by many commentators, both ancient and modern. Is it not revealing that the whole passage on the perfection of the decad in the Peripatetic "Problems" (910 b 23–911 a 4) also refers to the Pythagoreans, and not to Speusippus, although it is derived almost *verbatim* from his book, whereas the specific belief ascribed here to the Pythagoreans comes from the Academic interpretations of "Timaeus"[63]?

Before going into details of Speusippus' theory of the decad, we should bear in mind the following: 1) before Aristotle, neither the acousmatic nor the philosophical tradition of the Pythagorean school says anything on the perfection of the decad; 2) Aristotle more often connects this idea with the Platonists than with the Pythagoreans; 3) the Early Academy developed a specific metaphysical theory on the number ten, an arithmological part of which was set forth by Speusippus in the work "On the Pythagorean numbers".

There is no doubt that Speusippus did use some Pythagorean material and ideas[64], but in view of everything said so far we can expect that the doctrine on the perfection of the decad represents his own paramathematical teaching[65]. It is very likely that it was Speusippus who made a decisive step in the identification of early Pythagorean τετρακτύς with the number ten[66]. Indeed, after him τετρακτύς has been interpreted not simply as "tetrad", i.e. as the number four or a group of the first four numbers (1,2,3,4), but as something wholly unusual, namely as "a group of the first four numbers, the sum of which equals 10"[67]. Such a meaning is clearly based on Speusippus' theory and could not be the original one, as all the words formed along the same rule show[68]. τετρακτύς does not much differ from τετράς, it "contains" or "embraces" the numbers from 1 to 4, but not their sum. It is not coincidental that most of the ancient authors use τετράς and τετρακτύς interchangeably when quoting the Speusippian

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[63] T.L. Heath. *Mathematics in Aristotle* (Oxford 1949) 258 f.; Flashar, 567 f.; cf. Tarán, 274 f.

[64] The notions of linear, polygonal, surface and solid numbers that Speusippus discusses in the first half of his book, are derived from early Pythagorean arithmetic, but he uses them for his own purposes (Tarán, 262 f., 298).

[65] Tarán, 259 f., 269 ff., 275 ff.; Huffman. *Philolaus*, 361 ff.

[66] Frank, 260 n. 1; cf. Burkert, 72, 186 f., Tarán, 273 f. The evidence on τετρακτύς see: A. Delatte. *Études sur la littérature pythagoricienne* (Paris 1915) 249 ff.

[67] Delatte (254 f.) interprets it now as "ensemble de quatre premiers nombres", now as "somme des quatre premiers nombres", Burkert (72) as "four-group", made up of the numbers, 1,2,3,4, which add up to 10".

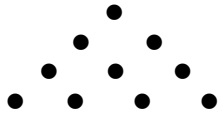
[68] For example, τρικτύς means "sacrifice consisting of three animals", πεντηκοστύς "body of fifty" in the Spartan army, etc. Cf.: "τετρακτύς 'Zahl 4' Pythag. wohl erst nach τρικτύς" (E. Schwyzer. *Griechische Grammatik*. Bd. I. [München 1990] 596 f.) and *LSJ*, s.v. τετρακτύς: "Pythagorean name for the sum of the first four numbers, i.e. 10".

"formula", and it is τετράς, and not τετρακτύς which we find in Speusippus' own wording (fr. 28 Tarán). This suggests again that this "formula" is not based on the authentic Pythagorean tradition.

As Delatte rightly pointed out, originally τετρακτύς derives from the early Pythagorean harmonics; our primary source, a Pythagorean acousma, identifies it with ἁρμονία: τί ἐστι τὸ ἐν Δελφοῖς μαντεῖον; τετρακτύς· ὅπερ ἐστὶν ἡ ἁρμονία, ἐν ᾗ αἱ Σειρῆνες (Iambl. *VP* 82). The most natural interpretation of τετρακτύς as a harmony is then that it comprises all συμφονίαι, or contains the numbers of the basic harmonious intervals, octave (2:1), quinte (3:2) and quarte (4:3)[69]. Precisely these four numbers were important for the Pythagoreans, whereas the number ten did not play any role either in their harmonics[70] or in their arithmology[71]. Speusippus, however, was not interested in the number four as such, much more attractive to him were the various correspondences between tetrad and decad, to which he returns again and again:

> For 1 is a point, 2 is a line, 3 is a triangle and 4 is a pyramid; all these are elements and principles of the figures like them. In these numbers is seen the first of progressions… and they have 10 for their sum. In surfaces and solids these are the elements: point, line, triangle, pyramid. The number 10 exhibits them and possesses perfection. For 4 is to be found in the angles or faces of a pyramid, and 6 in the sides, so making 10… In the case of solids, you would find this property also, by going up to four, so that the decad is reached in this way also… (fr. 28 Tarán).[72]

The best way to visualise an intimate connection between four and ten is to draw an equilateral triangle or the so-called triangular number, which admittedly leads us to the early Pythagorean pebble arithmetic[73]. To be sure, the properties of the triangular, oblong, square and other *classes* of numbers were treated in this arithmetic, but it did not single out this particular triangular number, for there is nothing special in it from a mathematical point of view. Yet to illustrate that four "generates" ten, Speusippus had a serious reason to be interested exactly in this figure, since for him an equilateral triangle is the "first" among the plane figures, while a pyramid, based on this triangle, is the "first" among the solids. All this, of course, is mathematically meaningless, especially if we take into account that in Speusippus' view an equilateral triangle has "in a certain sense" only one side and one angle, because they are equal, and equal is always undivided and of the same kind. He was obviously inspired not so much by



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[69] πάσας δὲ τὰς συμφονίας περιέχει ἡ τετρακτύς. συνέστησε μὲν γὰρ αὐτὴν α΄ καὶ β΄ καὶ γ΄ καὶ δ΄. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἔστιν ἥ τε διὰ τεσσάρων συμφονία καὶ ἡ διὰ πέντε καὶ ἡ διὰ πάσων (Theon Smyrn. p. 58.13 f. Hiller).

[70] It is noteworthy that Eudemus of Rhodes, being the best expert in the Pythagorean science, speaks in this connection about the number nine, and not ten: "The relations of the first three harmonious intervals are in the limits of the first nine numbers, for the sum of 2, 3 and 4 equals to 9" (fr. 142 Wehrli).

[71] "Primitive" as opposed to the "learned" arithmology does not single out the number ten as significant, the most important numbers here are 2, 3, 4, 5, 7 and 9.

[72] See also the argument in the pseudo-Aristotelian "Problems" that very likely also derives from Speusippus: "Is it because ten is the beginning of number, since ten is produced by adding one, two, three, and four?" (910 b 34).

[73] Lucian. *Vit. auct.*, 4; cf. Nicom. *Intr. arithm.* II, 8; Delatte, 256 n.1, Burkert, 72. Although we do not know whether Speusippus drew this figure, it seems quite probable, since it was very convenient for illustrating his favourite idea that line is generated by point, plane by line, and solid by plane (on this derivation see: Burkert, 66 f.).

the Pythagorean mathematics, as by "Timaeus" (54a–d), where the constitutive elements are constructed of the same three kinds of triangles, as mentioned by Speusippus (the equilateral, the right-angle isosceles, and the half-equilateral)[74].

Thus, Speusippus and especially his work "On the Pythagorean numbers" is the most likely source both for the later identifying of τετρακτύς with the decad and for the general theory on its perfection. Even if he had not directly attributed these ideas to the Pythagoreans, the very title of his work paved the way for such an interpretation. Though Aristotle usually distinguished between Pythagorean and Academic theories[75], he was not and could not be consistent in this respect, especially considering his sources and his aim to represent the Pythagoreans as the philosophical predecessors of the Academy[76]. Relying only on Aristotle's account, we are not always in the position to discern what is Platonic and what Pythagorean, and I am not sure that he himself was able and eager to bear this distinction constantly in mind. Here are only two examples of a possible confusion.

1) Although Aristotle assigns the table of the ten opposites to some group of the Pythagoreans, what we actually have here is "a continuous transition between Pythagorean and Platonic"[77]. Aristotle twice connects the table with Speusippus (*Met*. 1072 b 30 f.; *NE* 1096 b 6); after the first two pairs (πέρας–ἄπειρον, περιττόν–ἄρτιον) important both for the Pythagoreans and for Plato the third place occupies the pair ἕν–πλῆτος, which is a cornerstone of Speusippus' philosophy[78], and the fifth ἄρρεν–θῆλυ, attested among Xenocrates' principles (fr. 213 Isnardi Parente). The sixth and ninth pairs (rest and movement, good and bad) are, according to Aristotle, typically Platonic (*Met*. 1084 a 32 f.), whereas such opposites as cold–hot and moist–dry, which were really important for the Pythagorean thinkers[79], are conspicuously absent in the table. What Aristotle says about the table in his work "On the Pythagoreans" (καὶ τῇ δεκάδι συμβολικῶς ὡς τῷ παντὶ ἀριθμῷ συμπληρώσαντες ἑκατέραν ἑκάστην ἀντίθεσιν τῶν δέκα οὕτω παρέλαβον ὡς πάσας τὰς ἑαυτῆς συγγενείας συνεμφαίνουσαν)[80] also points rather to Speusippus than to the Pythagoreans[81]. Between the opposites of

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[74] Tarán, 285 f. There is one more line leading to the "Timaeus", namely, to the famous "number of the soul" constructed of two "tetractys", or of two number progressions: 1,2,4,8 and 1,3,9,27 (*Tim*. 35b, cf. Theon. Smyrn. p. 94.10 ff. Hiller). The last number of this series is equal to the sum of all the preceding numbers: 1+2+3+4+8+9 = 27, which seems as elegant as 1+2+3+4=10. On the "Platonic" tetractys as the number 36 see: Delatte, 255 f.

[75] Burkert, 15 f., 28 ff., 79 f., 230 ff.

[76] There is an obvious tendency in Aristotle to underline Plato's affinity to the Pythagoreans and even his dependence on them (*Met*. 987 a 29 ff., 987 b 11 f., 987 b 22 f., 990 a 30, 996 a 6, 1001 a 9, 1053 b 12; *Phys*. 203 a 6 f.), which was further developed by the Peripatetics (Dicear. fr. 41 Wehrli, Eud. fr. 31, 60 Wehrli, Aristox. fr. 131 Wehrli). See: Cherniss, 386 ff.; W.A. Heidel. The Pythagoreans and Greek Mathematics // *AJP* 61 (1940) 10 f.; Zhmud. *Wissenschaft*, 268 ff.

[77] Burkert, 51.

[78] Arist. *Met*. 1092 a 35, 1087 b 4, b 25, 1085 b 5; Cherniss, 391; Tarán, 33 ff.

[79] See: Alcmaeon (24 B 4), Menestor (32 A 3–5, 7), Hippon (38 A 11), Philolaus (44 A 27).

[80] Simpl. *In De caelo*. p. 386.9 ff. = fr. 164 Gigon. συμβολικῶς is late, so we have here rather paraphrase than exact quotation.

[81] Cf. on the decad (fr. 28 Tarán): παράδειγμα παντελέστατον τῷ τοῦ παντὸς ποιητῇ θεῷ; ἔτι πάντες οἱ λόγοι ἐν τῶι ι'; on the sequence point–line–triangle–pyramid: ταῦτα δὲ πάντα ἐστὶ πρῶτα καὶ ἀρχαὶ τῶν καθ' ἕκαστον ὁμογενῶν. On the role of τὰ ὅμοια and the principle of ὁμοιότης in Speusippus' philosophy see: Tarán, 64 f., 393 f. He wrote two special writings on this subject: "On similar things" (incidentally – in *ten* books!) and "Divisions and suppositions concerning similar things" (D.L.

the Pythagoreans, e.g. of Alcmaeon (cold–hot, bitter–sweet, 24 B 4), there is no συγγενεία; as for the Pythagorean "number of the whole", elsewhere Aristotle attests that it is three, and not ten[82]. Therefore, however much in its detail the table ultimately derives from the Pythagorean tradition, in its final form of the *ten pairs of distinct kindred* (*or similar*) *opposites* it is a product of the Academic systematisation, and exactly this form and not the opposites themselves were of interest to Aristotle[83]. But if there is nothing in the table, for which *we* could not find an immediate correspondence in Academic metaphysics, what made Aristotle believe that it is Pythagorean?

2) Equally confused is the issue of number symbolism. Aristotle frequently refers to the three Pythagorean "number definitions" (δικαιοσύνη is four, γάμος is five, καιρός is seven)[84], which look quite authentic. In his treatise "On the Pythagoreans" he evidently treated this issue more extensively, for Alexander derives from it some other definitions: 1) νοὺν δὲ καὶ οὐσίαν ἔλεγον τὸ ἕν, *nous* διὰ τὸ μόνιμον δὲ καὶ τὸ ὅμοιον πάντῃ καὶ ἀρχικόν, and *ousia* ὅτι πρῶτον ἡ οὐσία. 2) δόξαν δὲ τὰ δύο διὰ τὸ ἐπ᾽ ἄμφω μεταβλητὴν εἶναι· ἔλεγον δὲ καὶ κίνησιν αὐτὴν καὶ ἐπίθεσιν[85]. Now, these definitions sound neither archaic nor Pythagorean: we can compare τὸ ὅμοιον πάντῃ with Speusippus' indefatigable search for τὰ ὅμοια[86], or remember that Xenocrates identifies νοὺς with τὸ ἕν (fr. 213 Isnardi Parente) and discerns *two* kinds of δόξα, true and false (fr. 83 Isnardi Parente), which has an obvious parallel in the "Pythagorean" definition. οὐσία is a Platonic–Peripatetic term; according to Aristotle, Plato identified it with the One (τὸ μέντοι γε ἕν οὐσίαν εἶναι, *Met*. 987 b 21); in his later dialogues Plato contrasts οὐσία with κίνησιν (*Soph*. 232c, *Leg*. 966e), whereas Eudemus asserts that he identified κίνησις with "great and small", i.e. with *aoristos dyas* (fr. 60 Wehrli)[87].

Therefore, we again find in the Academic tradition all the material needed for the construction of the "Pythagorean" definitions. To be sure, the other Academic theory defines ἐπιστήμη as two, and δόξα as three[88], but there are too many coincidences between "Pythagorean" and "Academic" views to make us believe that it was possible for Aristotle to draw a sharp distinction between them. There is no doubt that Platonic arithmology was partly based on the Pythagorean; it is even possible that some of the older "number definitions" have been reformulated later by the Platonists[89]. But we can hardly imagine that Aristotle had in his

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IV, 5). Tarán (348 f.) does not see any connection of the table with Speusippus; cf. Frank, 255; Cherniss, 241 n. 11, 391.

[82] *De Caelo*. 268 a 10 ff. See below, p. 20.

[83] *EN*. 1096 b5, 1106 b 29; cf. his criticism of Alcmaeon who has not defined, how many and of what kind are the opposites (*Met*. 986 a 26 f.). For Xenocrates one could reconstruct the following table of opposites: μονάς–δυάς, ἄρρεν–θῆλυ, Ζεύς–μήτηρ θεῶν (Rhea?), περιττόν–ἄρτιον, νοῦς–ψυχή (fr. 213 Isnardi Parente).

[84] *Met*. 985 b 29–30, 990 a 23, 1078 b 22–23, *EN* 1132 b 23, *MM* 1182 a 11.

[85] Alex. *In Met*. p. 39.16–22 = fr. 162 Gigon.

[86] See above, n. 81.

[87] General association of one with rest and two with motion derives from Plato's development of the Eleatic doctrine; cf. the two pairs in the table, ἕν–πλῆτος, ἠρεμοῦν–κινούμενον.

[88] Arist. *De an*. 404 b 16 ff.; Burkert, 25 f., 70.

[89] Aristotle illustrates the significance of the number seven in three different ways (Alex. *In Met. comm*. p. 38.16–39.8 = fr. 162 Gigon): the first explanation is probably based on Hippon (38 A 16, cf. 24 A 15),

hands an authoritative textbook of Pythagorean arithmology, which he could compare with the writings of the Platonists. So, if he called something "Pythagorean" which appears to us to be rather Platonic, it means that he not always had at his disposal unambiguous criteria for distinguishing between them.

This is confirmed by the fact that he failed to hand them down to his students, otherwise Theophrastus would not have ascribed to the Pythagoreans an Academic theory of *monas* and *aoristos dyas* (*Met*. XI a 27 = 58 B 14), and the Peripatetic author of the "Problems" the Speusippian theory of the decad[90]. Hence, there is no wonder that we encounter in the Peripatetic doxography on Pythagoras both the theory of *monas* and *aoristos dyas* and the theory of the decad[91]. According to the doxographer Pythagoras said that "the potentiality (δύναμις) of the decad is in four and in the tetrad", and that is why the Pythagoreans even swore by the tetrad (*Dox*., p. 282.7–8):

> οὐ μὰ τὸν ἁμετέρᾳ κεφαλᾷ παραδόντα τετρακτύν
> παγὰν ἀενάου φύσεως ῥίζωμά τ᾽ ἔχουσαν.

Should we really imagine that the early Pythagoreans understood τετρακτύς in an Aristotelian way, i.e. as ten δυνάμει[92]?

The famous "Pythagorean oath" deserves, perhaps, a small digression. As a source it is of very doubtful origin as, indeed, is everything that alludes to the alleged Pythagorean secrecy[93]. While the acousma about τετρακτύς certainly belongs to those treated by Aristotle[94], the oath appears for the first time in Aëtius[95]. The Doric dialect never occurs in the acousmata and points rather to Hellenistic pseudo-Pythagorean poetry (φύσεως is Attic) than to the Doric prose of Philolaus and Archytas. Avoiding swearing by the names of gods is quite a common

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the second on Philolaus' astronomical system, and the third is very close to Academic arithmology, especially to Speusippus (fr. 28 Tarán).

[90] Flashar (568 f.) suggests that he was following Theophrastus.

[91] *Dox*., p. 281.6 ff. = 58 B 15. The possible authors of the latter theory are considered to be Xenocrates (Tarán, 274 f.) and Speusippus (Flashar, 567 f.; Burkert, 73 n.122). Cf. also: οἱ Πυθαγόρειοι τοὺς ἀπὸ μονάδος ἄχρι δεκάδος ἀριθμοὺς ἀρχὰς ἔλεγον ἢ τὰς δέκας συστοιχίας (Theophr. fr. 3 Diels = fr. 227A FHSG).

[92] Cf. Philon. *De plant. Noë*, 123: καλεῖται δ᾽ ἡ τετρὰς καὶ "πᾶς", ὅτι τοὺς ἄχρι δεκάδος καὶ αὐτὴν δεκάδα περιέχει δυνάμει. A further development of this idea we find in Sextus Empiricus (*Adv. math*. IV,3; VII, 94–95): rejecting any potentiality, he takes τετρακτύς as the actual sum of the first four numbers, i.e. as the perfect number ten.

[93] See: Zhmud. *Wissenschaft*, 85 ff. An oath in verse is suspicious in itself, since the examples of the usage of such an oath by any real political, social or religious society are unknown, at least in the classical and Hellenistic periods (suggested by A. Zaicev).

[94] Delatte, 271 ff., Burkert, 166 ff.

[95] Delatte, 253 thought that it was known to Timaeus of Tauromenium, but this is impossible to prove. Burkert also relates it to early Pythagoreanism, although not very consistently. On the one hand, he says (186 n. 155) that the second line of the oath could not be older than the second half of the 5th century, so the first line possibly stood by itself in the beginning, and on the other (72), that Xenocrates called matter ἀέναος (fr. 102 Isnardi Parente) and it is tempting to believe that he knew the verse on the tetractys (this was a hypothesis of Zeller). Now Delatte called this hypothesis "peut-être trop hardie", it was refused by the two editors of Xenocrates, R. Heinze and M. Isnardi Parente, whereas Burkert does not mention Timaeus as a possible source. J. C. Thom (*The Pythagorean Golden Verses* [Leiden 1995] 37 f., 171 ff.) also believes that the oath is early Pythagorean.

motive in pseudo-Pythagorica[96]: at the beginning of his treatise Φυσικός Pythagoras himself swears not by the gods but by air and water (perhaps, in order to give an example to his followers), and incidentally in the same negative form (οὐ μὰ τὸν ἀέρα…) as in the oath[97]! That is why, says Iamblichus (*VP* 150), the Pythagoreans swear not by the gods, but by Pythagoras, although even him they do not call by name.

Before leaving the issue of number symbolism, it is worth noting that Aristotle too was not completely free from the "Pythagorean" tendency of assigning importance to some prominent numbers, a tendency that is in fact deeply rooted in human psychology. So, for example, when describing the colours of the rainbow (*Meteor*. 372 a 1–11), he asserts that they are only three, i.e. red, green and blue; further he notes:

> No further change is visible, but three completes the series of colours (as we find three does in most other things), and the change into the rest is imperceptible. Hence also the rainbow appears with three colours (374 b 32 f.).

To reduce seven colours of the rainbow to three Aristotle obviously violates the φαινόμενα: he reduces red and orange to one colour; blue, dark blue and violet also to one, further he treats green as a primary colour (although it can be produced by mixing yellow and blue) and the appearance of the yellow in the rainbow as an optical illusion, since "the rainbow is by nature (ἀνάγκη) three-colour". In *De caelo* Aristotle comes out even more energetically on behalf of the fundamental significance of the number three, referring to no one else but the Pythagoreans! Every body, says he, is given in three dimensions, and there are no other dimensions except them:

> For, as the Pythagoreans say, the world and all that is in it is determined by the number three, since beginning and middle and end give the number of an "all", and the number they give is the triad. And so, having taken these three from nature as (so to speak) laws of it, we make further use of the number three in the worships of the Gods… And in this, as we have said we do but follow the lead which nature gives (268 a 10 ff.).

It turns out, therefore, that the number principle is not some Pythagorean fantasy, as one could judge from those passages where Aristotle criticises the number speculations of this school, but a fundamental natural regularity that we have to imitate in our life and follow in our research. Of course, from the fact that all (or many) things exist by three, it does not follow for Aristotle that all *is* three or that all *is* number[98]. There are no grounds to believe, however, that this conclusion, which Aristotle constantly ascribes to the Pythagoreans, really was made by them.

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[96] See, e.g. the second speech of Pythagoras (Iambl. *VP* 47 = Thesleff, p. 180.25 f.) and Anonymus Diodori (Diod. X,9.1 = Thesleff, p. 232.26 f.), cf. D.L. VIII, 22; Iamb. *VP* 144, 150.

[97] D.L. VIII,6 = Thesleff, p. 170.17 ff. "With its negative formulation, the oath probably applied primarily to the secrecy of Pythagorean doctrine" (Burkert, 187).

[98] Cf. Ion of Chios: πάντα τρία καὶ οὐδὲν πλέον ἢ ἔλασσον τούτων τῶν τριῶν (36 B 1).